Verification Example 1 – Random Applied Stress (SI Units)

Introduction

This example problem is used to verify the results of Darwin™ for the stress scatter random variable. Consider the semi-infinite block shown in Figure 1. A circular crack with initial area 0.20296 mm^2 is located in the geometric center of plane A. The cross-section where plane A intersects the part is 2.54 m x 2.54 m.

The block is subjected to a lognormally distributed applied stress with median = 689.48 MPa and a coefficient of variation (COV) = 10%. The target service life of the block is 20,000 cycles. No inspections are performed, and the life scatter is modeled as a deterministic variable with value equal to 1.0.

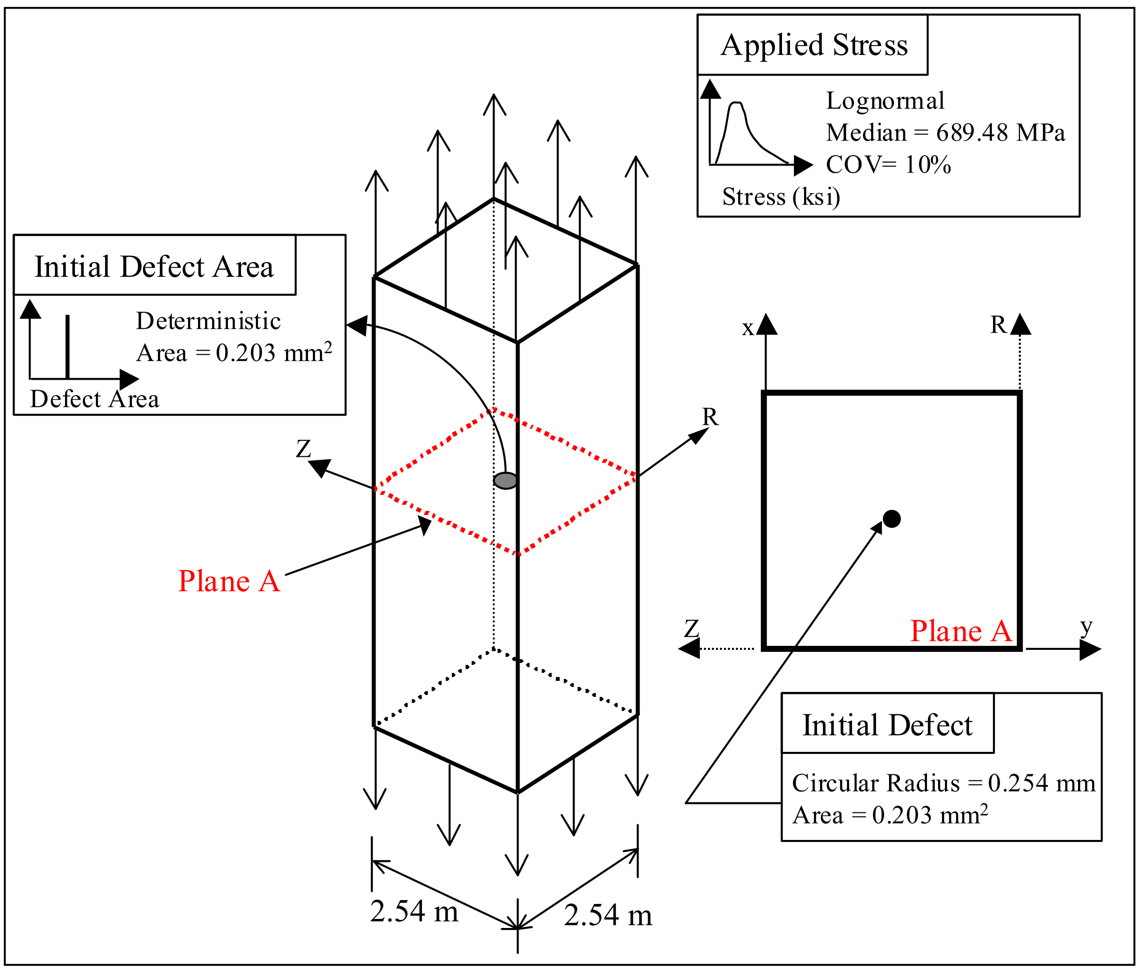

Figure 1. Schematic for Example #1

Darwin™ Solution

The fatigue failure probability is defined as the probability of violating the fatigue limit state g(**X**,**Y**,t). Fatigue failure occurs when the stress intensity factor K exceeds the fracture toughness K_c:

$$g(\boldsymbol{X},\boldsymbol{Y},t) = K_c - K(\boldsymbol{X},\boldsymbol{Y},t) \leq 0$$

where **X** is a vector of input variables unrelated to inspections, **Y** is a vector of input variables related to inspections, and t is flight hours. A negative or zero value of g(**X**,**Y**,t) represents a failure event.

This problem is solved using Monte Carlo simulation. The results are indicated in Table 1. It can be observed that the result is dependent on the number of Monte Carlo samples selected.

Table 1. Summary of Darwin™ Results

Number of Samples	**Probability of Failure**		
	Lower Bound (95% Confidence)	Mean	Upper Bound (95% Confidence)
100	8.100×10^{-6}	9.643×10^{-6}	1.119×10^{-5}
1000	1.007×10^{-5}	1.054×10^{-5}	1.102×10^{-5}
10,000	1.081×10^{-5}	1.096×10^{-5}	1.111×10^{-5}
100,000	1.091×10^{-5}	1.095×10^{-5}	1.100×10^{-5}

Analytical Solution

Applied stress is the only random variable for this problem. The probability of failure is equal to the probability that applied stress is greater than or equal to S_{crit}, where S_{crit} is the stress at which life is 20,000 cycles (see Figure 2).

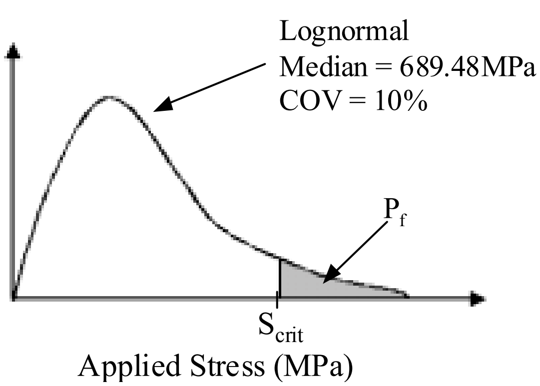

Figure 2. Analytical failure probability prediction

S_{crit} can be computed using the deterministic computation capability of Darwin™. The lives associated with several applied deterministic stress values are indicated in Table 2. It can be observed that an applied stress value of 657.76 MPa provides a life nearly equal to 20,000 cycles. Therefore, $S_{crit} \approx 657.76$ MPa.

Table 2. Determination of the critical stress

Critical Stress (MPa)	620.53	655.00	656.38	657.38	659.14	660.52	661.90	689.48
Life (cycles)	25,171	20,343	20,187	20,015	19,843	19,687	19,531	16,609

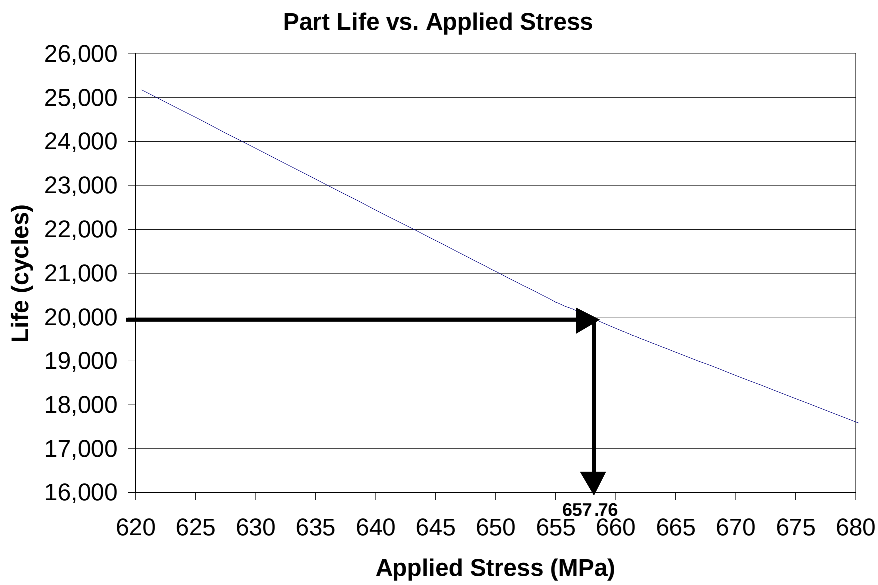

Figure 3. Determine the applied stress associated with the target service life

The probability of failure for the block is:

$$P(\text{failure}) = P_{f|d} \cdot P_d$$

where: $P_{f|d}$ = Probability of a failure given that a defect has occurred

P_d = Probability of occurrence of a single defect

$$P_{f|d} = P(S > S_{crit}) = P(S > 657.76 \text{ MPa})$$

where S = lognormal (median=689.48 MPa, COV = 0.10)

$$P_{f|d} = 1 - \Phi\left[\frac{\ln(S_{crit}) - \lambda}{\zeta}\right]$$

where: $\lambda = \ln(\text{median}) = \ln(100)$

$\zeta^2 = \ln(1 + COV^2) = \ln(1 + 0.1^2)$

$$P_{f|d} = 1 - \Phi\left[\frac{\ln(657.76) - \ln(689.48)}{\sqrt{\ln(1+0.1^2)}}\right] = 0.681569$$

$$P_d = \left(\frac{\text{Expected \# of defects}}{10^6 \text{ lbs titanium}}\right)(\text{zone volume})(\text{titanium density})$$

$$= \left(\frac{0.0002204 \text{ defects}}{10^6 \text{ kgs titanium}}\right)\left(16.387064 \text{ m}^3\right)\left(\frac{4450 \text{ kg}}{\text{m}^3}\right)$$

$$= 1.61 \times 10^{-5}$$

$$P_f = (0.681569)\left(1.61 \times 10^{-5}\right) = 1.097 \times 10^{-5}$$

Verification

Comparing the Darwin™ results at 100,000 samples to the analytical solution results, it can be observed that the analytical solution falls within the 95% confidence bounds for the probability of failure.

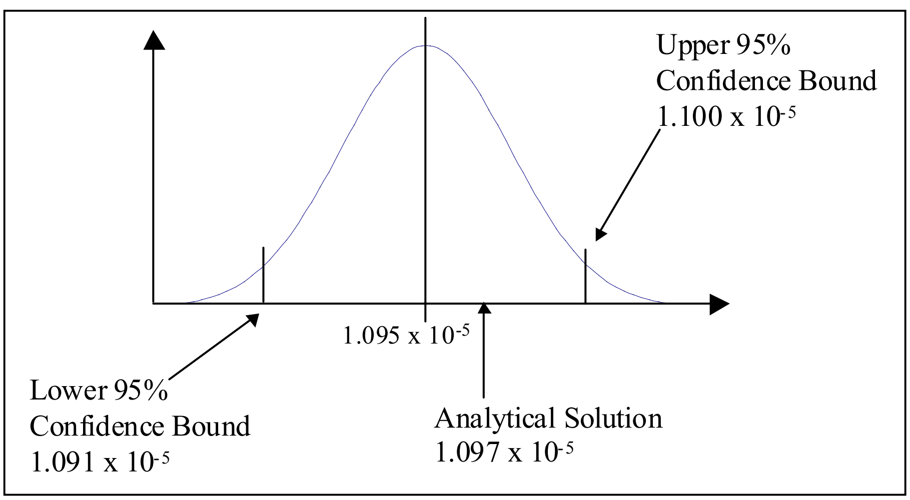

Figure 4. Comparison of the Darwin™ results to analytical results

Input Data Files

Ex_1_SI.dat

```
*darwin         !    header line

*TITLE
     Darwin Verification Example 1
     Infinite Plate - Embedded Crack
     Appendix A Prog Rep No 10
     SwRI - Feb 2001
     Units = MPa

*Fracture
 flight_life

*PROB_METHOD
  Method
   Monte

  Seed
 185465.0

  Zone_number    Samples
      1           100000


*UNITS
METRIC

*ANALYSIS OPTION
3

!--------------Stress File Data----------
*STRESS INPUT

! Number of Missions =      1
 Mission  1  !     File Name                Load Case
                    Example1_SI         rsf            1

FAILURE toughness FACTOR
     1.0

stress multiplying factor
   1.0  0.1      ! median, cov


!----Service Life Checks----
*SERVICE LIFE
20000,20000,1000

!------Defect Distribution----------

*DEFECT INPUT
DEFECT DISTRIBUTION 1
Area     Exceedance
0.20296088  0.0002204
0.20296733  0.00002204


!------ MATERIAL PROPERTY INPUT -------

*MATERIAL INPUT
```

```
!------- Air paris, no mean stress -----------------------
!------- Vacuum paris, Walker eqn. mean stress effects -----
Material 1

DADN DATA
AIR
FCG_Format  paris
Stress_ratio_Format    None
Temperature_Interpolation_Format   Next_Highest

! C                n         kth    kc          Temp
9.255629E-13      3.87       0.0    64.50211   37.7778

vacuum
FCG_Format                         paris
Stress_ratio_Format                None
Temperature_Interpolation_Format   Next_Highest

! C                n         kth    kc          Temp
9.255629E-13      3.87       0.0    64.50211   37.7778

STRESS_STRAIN_MONOTONIC
Input_Format     Ramberg-Osgood
Temperature_Interpolation_Format   interpolate

!epsilon0   sigma0    alpha   n    yield     temp
0.008647    1013.542   1.0   30.0  965.266   21.11
0.008009     938.745   0.99  30.0  896.318   93.33
0.009013     869.949   0.99  30.0  827.371  315.56
0.008754     724.254   0.99  30.0  689.476  648.88


STRESS_STRAIN_CYCLIC
Input_Format    Ramberg-Osgood
Temperature_Interpolation_Format   closest

!epsilon0   sigma0    alpha   n    yield     temp
0.008647    1013.542   1.0   30.0  965.266   21.11
0.008009     938.745   0.99  30.0  896.318   93.33
0.009013     869.949   0.99  30.0  827.371  315.56
0.008754     724.254   0.99  30.0  689.476  648.88

!---------------------------------------------------
! Zone Dependent Data
*ZONE DATA

ZONE      1
     Elements
        1
!                            median   cov   (assumes lognormal)
     life_scatter              1.0    0.0
     Volume Multiplier         1.0
     Defect Distribution       1
     Material                  1
     Crack Type                EC02
     Crack Plane               hoop
     Crack Stress              0.
     Plate Dimensions
      1.27    2.54       1.27        2.54
     Crack Location           1.27        1.27

*END
```

Example1_SI.rsf

```
1
1 16.387064
1 1 7 7
1.27   0.0   0.508  1.016  1.524  2.032  2.54
1.27  1.27  1.27   1.27   1.27   1.27   1.27
0.5 0.0  0.2  0.4  0.6  0.8  1.0
0.5 0.5  0.5  0.5  0.5  0.5  0.5
1.00000E-20
689.4757  689.4757  689.4757  689.4757  689.4757  689.4757  689.4757
     0.0     0.0     0.0     0.0     0.0     0.0     0.0
689.4757     0.0     0.0     0.0     0.0     0.0     0.0
     0.0     0.0     0.0     0.0     0.0     0.0     0.0
```

Summary of Output Data

Probability of Failure (Darwin™ Solution)

```
                    :
==========================
       Disk Summary
==========================

Expected Number of Defects (all sizes) Per Disk
                1.61000E-05
                    :
--------------------------------------------------
      95% Confidence Bounds at  20000 cycles
          0.00  % of disks not inspected
             (unconditional results)
--------------------------------------------------

             pf without inspection                  pf with inspection
Zone  lower bound    mean    upper bound   lower bound     mean    upper bound
----  ----------------------------------   ----------------------------------
  1     1.01E-05    1.06E-05    1.10E-05      1.01E-05    1.06E-05    1.10E-05
----  ----------------------------------   ----------------------------------
disk    1.01E-05    1.06E-05    1.10E-05      1.01E-05    1.06E-05    1.10E-05
```

Deterministic Life (Analytical Solution)

```
                    :
******************************
     a vs. N Crack Growth

  22 Jan 2001     09:18:55
******************************
                    :
Total Life =     20015 Cycles
                    :
******************************
    Begin Risk Computation

  22 Jan 2001     09:18:56
******************************
```

Appendix – Unit Conversion Information

	US units	US to SI Multiply by:	SI to US Multiply by:	SI units
Defect Distribution	Defects per Million pounds	2.2046	0.45359	Defects per million kgs
Initial Defect Area	Square mils	0.00064516	1550.003	Square mm
Temperature	°F	1.8*(F-32)	0.556*C + 32	°C
Stress	Ksi	6.894757	0.14504	MPa
Paris Coefficient, C	Ksi^{-n}-$in^{(1-0.5n)}$	0.0254/(1.0988^n)	(1.0988^n)/0.0254	MPa^{-n}-$m^{(1-0.5n)}$
Toughness (& ΔK)	Ksi-$in^{1/2}$	1.09884344	0.91005	Mpa-$m^{1/2}$
Rotor Geometry	In	0.0254	39.37	m