ABSTRACT
This paper describes some of the new surface damage capabilities in DARWIN™, a probabilistic fracture mechanics software code developed to evaluate the risk of fracture associated with aircraft jet engine titanium rotors/disk. An initial framework is presented in which a graphical user interface (GUI) is used to explicitly define the stresses and temperatures at the crack location for several crack geometries. A summary of the approach used to develop new stress intensity factor solutions for these geometries is also presented, including selected validation results.

INTRODUCTION
The application of probabilistic methods to fracture-mechanics-based life prediction is becoming increasingly essential in the aerospace industry. A probabilistic methodology is particularly well suited to the efficient design of components subjected to rare, critical (i.e., life-limiting) events, because it allows the designer to adjust nominal component parameters to meet quantitative reliability requirements. Aircraft turbine rotors may be subjected to rare critical events (e.g., uncontained engine failures) due to the presence of metallurgical (e.g., hard alpha) and manufacturing (e.g., surface damage) defects that can occur during the manufacturing process. To account for these events, the Rotor Integrity Subcommittee (RISC) of the Aerospace Industries Association (AIA) has recommended adoption of a probabilistic damage tolerance approach to supplement the current safe life methodology.

The recommendation led to the development of a computer program called DARWIN™ (Design Assessment of Reliability With INSpection), being developed in collaboration with major engine manufacturers (General Electric, Honeywell, Pratt & Whitney, and Rolls-Royce). The program computes the probability-of-fracture as a function of the number of flight cycles, considering random defect occurrence and location, random inspection schedules, and several other random variables. Both Monte Carlo simulation and importance sampling are available for use in probabilistic life predictions. A fracture mechanics module, called Flight Life, is also incorporated into the code. In addition, a user-friendly graphical user interface (GUI) is available to handle the otherwise difficult task of setting up the problem for analysis and viewing the results.

Previous papers [1-6] have focused on the probabilistic methodology for predicting the failure probability associated with hard alpha defects. Recently, the code has been enhanced to include new predictive capabilities for surface damage. An enhanced GUI has been developed to accommodate user defined mission profiles, and work is continuing to address fully three-dimensional finite element models. New stress intensity factor solutions for surface, corner, and through cracks at holes and surface cracks in plates have been derived from extensive boundary element analysis. These new solutions employ weight functions to address accurately the nonlinear stress gradients common at stress concentrations and other

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surfaces. Risk calculation capabilities have been enhanced to provide failure probability predictions for individual features.

This paper describes some of the new surface damage features in DARWIN\textsuperscript{TM}. An initial framework is presented in which a graphical user interface (GUI) is used to explicitly define the stresses and temperatures at the crack location for several crack geometries. A summary of the approach used to develop new stress intensity factor solutions for several crack geometries (i.e., surface, corner, and through cracks at holes and surface cracks in plates) is also presented, including selected validation results.

**AN INITIAL FRAMEWORK FOR SURFACE DAMAGE-BASED RISK ASSESSMENT**

Previous releases have focused on risk assessment of titanium aircraft engine rotor disks with inherent (hard alpha) defects. Inherent defects can occur anywhere within a disk, so a zone-based risk integration methodology is used to account for this uncertainty [2,4]. The disk is discretized into a number of zones of approximately equal risk, and the total disk risk is approximately equal to the sum of the risks associated with the individual zones. Variability in defect size is addressed using established exceedance curves [7]. Under this methodology, stress and temperature information are provided for the entire disk for all of the load steps modeled based on results from finite element analysis.

In contrast, surface damage is present only on the exterior surfaces of a component and is often located at features (e.g., bolt holes) or along other machined surfaces. Additional crack geometries were introduced in DARWIN\textsuperscript{TM} 4.0 to model surface features as shown in Fig. (1). Currently, a feature-based methodology is used to estimate risk, in which the disk risk is approximately equal to the sum of the risks associated with the individual features. For a typical disk, the number of features associated with surface damage assessment is relatively small compared to the number of zones associated with inherent-defect-based assessment. In addition, the stress/temperature data associated with a surface damage assessment can be obtained at discrete locations (i.e., data are not required for the entire disk). A surface-damage-based defect distribution is currently under development by the Rotor Integrity Sub Committee (RISC) of the Aerospace Industries Association (AIA) to address defect size variability.

The initial framework for surface-damage-based risk assessment is shown in Fig. (2). The user defines a mission profile and crack geometry for each surface feature in the disk. Included in the mission profile definition are the stress, temperature, and stress gradient values at discrete time steps.

As shown in Fig. (3), the stress gradient can be defined explicitly by the user, or it can be extracted directly from a 2D finite element model. The number and spacing of gradient points can be adjusted prior to extraction.

![Figure 1](image1.png) In the DARWIN\textsuperscript{TM} 4.0 release, surface damage features are modeled as one of the following crack types: (a) surface crack, (b) surface crack at a hole, or (c) corner crack at a hole.
Figure 2. Initial framework for surface damage in DARWIN™ (4.0 release). A mission profile consisting of stresses and temperatures at the crack location (and associated stress gradients) is defined for each surface feature.

Figure 3. Enhanced stress gradient definition capabilities for surface damage. The stress gradient can be defined explicitly by the user, or it can be extracted directly from a 2D axisymmetric finite element model. The number and spacing of points along the gradient can be adjusted by the user.
A summary of the primary differences in the damage tolerance approach for inherent and surface-damage-based defects in DARWIN™ is indicated in Table 1. It can be observed that the list of differences is relatively small. In fact, most of the same computational tools can be applied to both defect types to assess the risk of fracture, which is a major advantage to this approach.

Table 1. Summary of the primary differences in the damage tolerance approach for inherent and surface-damage-based defects in DARWIN™

<table>
<thead>
<tr>
<th>Defect Type</th>
<th>Inherent</th>
<th>Surface Damage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Crack Type</td>
<td>Embedded, Surface, Corner</td>
<td>Surface, Surface at Hole, Corner at Hole</td>
</tr>
<tr>
<td>Stress &amp; Temperature Data</td>
<td>2D Finite Element Model</td>
<td>User Defined</td>
</tr>
<tr>
<td>Risk basis</td>
<td>Volume Based</td>
<td>Feature (Geometry) Based</td>
</tr>
</tbody>
</table>

A number of other enhancements were added to improve assessment of deterministic crack growth, such as a user specified initial defect aspect ratio, and the capability to specify the defect distribution in terms of initial crack area, length, or depth (among other enhancements). In addition, a crack animation feature was added (Fig. (4)) that allows the user to visualize crack growth.

DEVELOPMENT OF STRESS INTENSITY FACTOR SOLUTIONS FOR CRACKS AT HOLES

Earlier versions of DARWIN™, which were focused on inherent material anomalies, contained stress intensity factor (K) solutions for cracks in simple rectangular plates. Available geometries included elliptical embedded cracks, semi-elliptical surface cracks, quarter-elliptical corner cracks, and edge and embedded through cracks. However, the surface damage problem is currently focused on bolt holes, and so additional K solutions for cracks emanating from holes were required. Numerous K solutions for cracks at holes are available in the literature, but these solutions are generally for simple remote loads such as uniform tension, bend, or pin loading. In contrast, DARWIN™ requires the capability to address a general nonlinear stress gradient on the crack plane. Therefore, it was necessary to develop a new set of weight function solutions for the geometries of interest.

Weight Function Method

A weight function (also known as influence function or Green’s function) formulation enables the evaluation of stress intensity factors (SIFs) by integrating over the crack surface a product of the applied stress in the uncracked body with the weight function for the cracked structure. The analytical forms for weight functions can be derived for some simple geometries, but approximate weight function forms are used for...
complex configurations, especially with free surfaces. In DARWIN™ 4.0, a one-dimensional weight function is employed to determine the SIFs resulting from a stress gradient varying in only the crack depth direction (away from the hole).

The fracture mechanics modules in DARWIN™ 4.0 for a center surface crack (CSC) at a hole and a corner crack (CC) at a hole utilize the approximate weight function proposed by Glinka [8-10]. The weight function at the c-tip (the maximum depth position for the surface crack, or the tip on the plate surface for a corner crack) is

$$W_c = \frac{2}{\sqrt{2\pi}} \left[ 1 + M_{1c} \sqrt{\frac{\pi x}{c}} + M_{2c} \frac{x}{c} + M_{3c} \left( \frac{x}{c} \right)^{\frac{3}{2}} \right] \quad (1)$$

The weight function at the a-tip (the crack tip in the bore of the hole for either a surface or corner crack) is

$$W_a = \frac{2}{\sqrt{2\pi}} \left[ 1 + M_{1a} \sqrt{\frac{\pi x}{c}} + M_{2a} \frac{x}{c} + M_{3a} \left( \frac{x}{c} \right)^{\frac{3}{2}} \right] \quad (2)$$

The variable $x$ is the distance normal to the hole axis measured from the location where the crack emanates (the root of the hole). The parameters $M_{1c}$, $M_{2c}$, $M_{3c}$, etc. depend on the geometrical parameters and are defined by reference solutions.

At the c-tip, $M_{1c}$, $M_{2c}$, and $M_{3c}$ are defined by

$$M_{1c} = \frac{\pi}{\sqrt{2Q}} (4Y_0 - 6Y_f) - \frac{24}{5}$$

$$M_{2c} = 3$$

$$M_{3c} = 2 \left( \frac{\pi}{\sqrt{2Q}} Y_0 - M_{1c} - 4 \right) \quad (3)$$

and at the a-tip, $M_{1a}$, $M_{2a}$, and $M_{3a}$ are given by

$$M_{1a} = \frac{\pi}{\sqrt{4Q}} (30F_0 - 18F_f) - 8$$

$$M_{2a} = \frac{\pi}{\sqrt{4Q}} (60F_0 - 90F_f) + 15$$

$$M_{3a} = -(1 + M_{1a} + M_{2a}) \quad (4)$$

$Q$ is the shape factor for an elliptical crack approximated by

$$Q = \begin{cases} 1 + 1.464 (a/c)^{0.65}, & a/c \leq 1 \\ 1 + 1.464 (a/c)^{-1.65}, & a/c > 1 \end{cases}$$

and $F_0$, $F_f$, $Y_0$ and $Y_f$ are normalized SIFs or the reference solutions. $F_0$, $F_f$ are obtained at the a-tip, and $Y_0$ and $Y_f$ are at the c-tip. The subscripts identify the two associated reference loads on the crack surfaces: $0$ denotes uniform tension, and $f$ denotes a linearly decreasing bending stress $\sigma(x) = -x/c + 1$.

The stress intensity factors at the a- and c-tips, $K_{a,c}$, can thus be determined by direct integration as

$$K_{a,c} = \int_0^c W_{a,c} \sigma(x) dx \quad (6)$$

where $\sigma(x)$ is the uniaxial stress applied on the crack surface, and the integration is carried out from $x=0$ to $x=c$.

The weight function for a through crack at a hole in an infinite plate is analytically derived based on the cross sectional hoop stress distribution for a hole in an infinite plate subjected to remote tension given by [11], and the associated SIF equation for a through crack at a hole approximated by Schijve [12]. The crack opening stress is given by

$$\sigma_c(x) = \frac{1}{2} \left[ 1 + \frac{1}{(1+x)^2} \right] + \frac{1}{2} \left[ 1 + \frac{3}{(1+x)^2} \right] \quad (7)$$

where $\bar{x}$ denotes the normalized x coordinate with respect to the radius of the hole $R$, and the associated normalized SIF for remote tension is defined by

$$f_c(x) = \frac{1}{2} \left[ 1 + \frac{1}{2x^2 + 19x^2 + 539 + 1} \right] \left[ \frac{2n\pi}{1 + 2n\pi} \right] 1 + \frac{2n\pi}{(1+x)^2} \quad (8)$$

where $\xi$ symbolizes the normalized crack length, $c/R$ [13]. The weight function for a through crack at a hole under an arbitrary uniaxial stress variation, $\sigma(x)$, can then be derived as

$$w_c(\bar{x}, \xi) = \frac{1}{\sqrt{2n\pi}} \sum_{i=1}^r \beta_i (\xi) \left( 1 - \frac{\bar{x}}{\xi} \right)^{\frac{3}{2}} \quad (9)$$

where $r$ varies from 1 to 5, and $\beta_i$’s are tabulated as a function of crack depth $c$. The SIF is expressed in terms of an integral over the whole crack depth:

$$K(\xi) = \sqrt{R} \int_0^c \sigma(\bar{x}) w_c(\bar{x}, \xi) d\bar{x} \quad (10)$$

Note that finite thickness ($a/t$) effects are included explicitly in the weight function formulation for the surface and corner cracks. Finite plate width effects for all three geometries are not included in the DARWIN™ 4.0 solutions, but are being added in DARWIN™ 5.0 with engineering approximations.

**Reference Solutions**

Numerical methods were used to generate highly accurate reference solutions for the approximate weight function.
formulation. The reference solutions for two-dimensional cracks at holes were derived using the FADD3D fracture mechanics software, a general boundary element (BE) code for three-dimensional linear elastic fracture analysis [14]. FADD3D includes a novel crack-tip element that allows the computation of SIFs directly from the nodal information along the crack front. The BE approach uses fewer elements than finite element (FE) approaches, substantially reducing the modeling effort while maintaining exceptional accuracy [15,16]. To facilitate the use of FADD3D, SwRI developed supplemental procedures for pre- and post-processing.

Reference solutions were generated at a wide range of combinations of geometrical aspect ratios: $R/t = 0.25, 1, \text{and} 2$; $a/t = 0.1, 0.2, 0.5, 0.8, \text{and} 0.9$; and $a/c = 0.5, 1, 2.5, 5, \text{and} 10$. Figure 5 shows an example model for a corner crack at a hole. Nine-node double curved patch elements were used. A single model typically contained fewer than 1100 elements.

**Comparisons and Validation**

To verify the weight function formulation, the SIFs evaluated at the $a$- and $c$-tips by the weight function method using the stress gradient around a hole in a plate subjected to remote tension were compared with independent FADD3D results using the same meshes and the same nonlinear stress field. The comparisons were performed for all aspect ratio combinations considered. The differences were less than 1%.

These FADD3D results for uniform remote tension were further evaluated by comparison with the Raju-Newman FE solutions for cracks at holes under uniform remote tension [17]. Sample comparisons are shown in Fig. 6. Here $F(\phi)$ is the normalized SIF, $K/(\sqrt{\pi a/\phi})$. FADD3D and Raju-Newman results compared favorably in many cases. In those cases where significant inconsistencies were observed, FE computations using the commercial code FEA-Crack were performed as an independent check. To achieve similar resolution to FADD3D, FEA-Crack computations used at least 6000 elements. The FADD3D results were confirmed by the

Figure 5. Example FADD3D model ($a/c=1, a/t=0.8, R/t=1$). The detailed mesh is shown for the crack surface only.

Figure 6. Selected normalized SIF results from FADD3D, Raju-Newman, and FEA-Crack for center surface crack (a-b) and corner crack (c) at a hole in a plate under remote tension. $2\phi/\pi$ denotes the angular position around the crack perimeter. $2\phi/\pi = 0$ or 2 is the $a$-tip in the bore, and $2\phi/\pi = 1$ is the $c$-tip.
FEA-Crack results. Note that the older Raju-Newman FE solutions, which employed fewer degrees of freedom, are less accurate at free surfaces due to the relatively coarse meshes used.

FUTURE WORK

Additional work is currently underway to broaden the generality of these solutions in future versions of DARWINTM. Planned enhancements include the effects of an off-center surface crack, a finite width plate, and a hole that is off-center in the plate. A new weight function formulation is also being developed that will address a fully bivarient stress field in the crack plane. In addition, a capability for modeling surface damage based on 3-dimensional finite element geometry is currently under development.

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